Design of curved waveguides: the matched bend

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A new criterion for the optimum design of curved dielectric waveguides is proposed. The bends designed according to this model are named matched bends. In the matched bend, the suitable choice of both bending radius and bending angle reduces the total losses of the bend and avoids the leaky-mode excitation at the end of the bend. For a given angle, a discrete number of bending radii that satisfy the matched bend criterion can be analytically determined. With respect to the lateral offset, matched bends are more robust to both fabrication tolerances and wavelength and can be realized in every technology. The reduction of the leaky-mode excitation at the output of the bend is a fundamental property when two or more components are cascaded. Ghost images in the spectral response, cross talk, and asymmetries of the transfer function are successfully avoided. Some examples that use buried, rib, ridge, and diffused waveguides are presented and discussed.

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1. INTRODUCTION

Recently some of the authors presented an effective method for the determination of the bend-mode characteristics in dielectric waveguides. Such a method is based on the expansion of the bend modes into modes of the straight waveguide and gives a new explanation of the propagating mechanism that takes place in a bend. The method requires the determination of shape and phase constant of the first two modes of the straight waveguide, the fundamental one and the first leaky mode, and, with simple analytical formulas, allows one to calculate the phase constant, the bending-induced birefringence, the bent-mode shape distortion, and the coupling losses with a straight waveguide for any desired bending radius. The accuracy is excellent, and the computational costs are very low.

In this paper a new concept for the optimum design of the bent sections of integrated optics components is introduced. The bend designed according to this criterion is called the matched bend. Moreover, it is shown that by use of matched bends it is possible to avoid the transition losses and, even more important, the distortion of the field outgoing from the bend. The proposed criterion is useful for the design and the optimization of each bent section of integrated optic components, it is well physically based, and it is applicable to every waveguide technology. Moreover, a very simple technique, based on the coupled-mode theory in bent waveguides, that is useful to determine the phase constant of the first straight leaky mode is described.

Usually, a good design of a bent waveguide section is intended to limit the pure radiation losses and the transition losses between the straight and the bent waveguides. However, the reduction of the excitation of the second mode, which in a monomode waveguide is leaky, is another extremely important factor to ensure the correct behavior of the whole device. The presence of higher-order modes, in fact, can affect or even compromise the performance, for example, of a Y branch, a multimode interference (MMI) coupler, a star coupler, or simply another bend in cascade to the first one. Not properly designed bends may lead to the excitation of higher-order leaky modes in the plane of the bend. Such excitation must be avoided as they have a severe impact not only on the losses but also on the expected performance of the whole circuit. Typically, leaky modes can give ghost images in the spectral response and result in a degradation of both the cross talk and the uniformity of the transfer function. The modes under the cutoff, in fact, can propagate for large distances before they are radiated away, and they often cause undesired coupling or severe distortions of the whole transfer function. This effect has been observed in both weakly guided and strongly confined waveguides.

In the literature the suggested method for reducing the transition losses is to slightly misalign the straight waveguide with respect to the bent one. In general, this small lateral offset in the transition allows one to reduce the transition losses but not always the distortion of the field, which remains with a radiative part. Moreover, the offset is difficult to realize because of the high resolution required in the technological process, and it is more sensible to the realization tolerances. Matched bends, instead, are characterized by a low sensibility to both wavelength and physical dimensions and do not require any additional or particular technological processes. The concept of the matched bend can also be applied to multimode waveguides.

This paper is structured in six sections. In Section 2 we review the propagation in a straight–bend–straight structure according to the proposed bimodal representa-
Propagation in a bent waveguide

In their recent papers, the authors demonstrated that the performances of two types of power splitter is shown. In Sections 4 and 5 the matched bend and the matched S bend are treated in detail. It is shown that for a given bending angle a discrete number of bending radii exist that satisfy the matched bend condition; several numerical examples are also discussed. In Section 6 the general properties of the matched bend condition are discussed. In particular, the dependence on the wavelength and on the geometrical parameters of both the waveguide and the circuit is investigated; a comparison with the lateral offset, a well-known solution to reduce the transition losses, is investigated in detail; and, finally, the impact of a bend on the performances of two types of power splitter is shown.

2. Propagation in a Bent Waveguide

In their recent papers, the authors demonstrated that the modes of a bent waveguide can be seen as the linear combination of the first two modes of the straight waveguide: the fundamental and the first asymmetric modes in the plane of the bend. Similarly, the modes of the straight waveguide can be seen as the linear combination of the first two bent modes. This modelization is scalar, that is, TE (TM) straight (bend) modes describe TE (TM) bend (straight) modes, and it is valid in the case of low radiation losses. The first asymmetric mode is leaky in a single-mode waveguide.

To describe the propagation mechanism that takes place in a bend, we consider the structure shown in Fig. 1, which is composed of two straight waveguides connected by a constant-radius bent waveguide. According to the proposed model, when the fundamental mode $\varphi_1$ of the straight waveguide arrives at the first straight–bend junction A, it excites the two bend modes $\varphi_{b1}$ and $\varphi_{b2}$, which begin to propagate in the bend. The field at the junction A is just equal to $\varphi_1$, but it changes its shape along the bend because the two bend modes have different phase velocities. However, after a beat length $L_B$, defined as

$$L_B = \frac{2\pi}{\beta_{b1} - \beta_{b2}},$$

where $\beta_{b1,2}$ are the phase constants of the two bend modes, $\varphi_{b1}$ and $\varphi_{b2}$ return in phase, and the total field returns to be identical to $\varphi_1$. The new method of designing curved waveguides proposed in this paper is based on this observation. Clearly, Eq. (1) and the following discussion are valid, separately, for TE and TM modes, if the radiation losses of $\varphi_{b2}$ are not too much higher than those of $\varphi_{b1}$.

In the bend the two excited modes $\varphi_{b1}$ and $\varphi_{b2}$ propagate uncoupled as they are the bent modes. The total field, however, can also be seen as the linear combination of the two straight modes that, because of the bending, are coupled, exchanging power similarly to what happens in a directional coupler. In the following, a ridge waveguide 8.5 $\mu$m wide and 6 $\mu$m thick, with 4 $\mu$m of upper cladding and $\Delta n = 0.69\%$, is taken as a reference. The evolution of the intensity of both the first and the second straight TE modes along the structure of Fig. 1 is shown in Fig. 2 for a constant bending radius of $R = 1.5$ mm. The ridge waveguide is shown in the inset. The abscissa starts at junction A, where only $\varphi_1$ exists. The simulation has been carried out by use of the beam propagation method, and the propagating field along the bend is projected on the previously calculated field of the fundamental and first-order modes. In the figure the typical periodical power exchange between the two coupled modes is evident, and, after a propagating distance $L_B$, the leaky mode $\varphi_2$ vanishes, and all the power comes back to the fundamental straight mode except for a small radiative loss. The maximum power exchange, occurring at $L_B/2$, is equal to $d$ and depends on the type of waveguide and on its bending radius. Similar results are obtained for TM modes, as discussed in Section 6.

If the bending radius is too small, a higher number of straight modes are needed to describe the bending modes, and a more complex oscillating beating figure is obtained. In that situation, however, the radiation losses are too high, and after a beat length the straight mode $\varphi_1$ is not recovered. In other words, the bend is useless.

The oscillatory nature of the losses has been observed by other authors, but if the beating is attributed only to the first leaky mode, a simple and useful technique for the design of the bent sections can be obtained. Figure 2 suggests that, under the hypothesis of small radiation losses, the propagation in a bent waveguide can be mod-

Fig. 1. Straight–bend–straight geometry.

Fig. 2. Beating of the first two straight modes in a monomode bent waveguide and definition of the beat length $L_B$ and of the beat depth $d$. The bending radius is $R = 1.5$ mm. In the inset the considered waveguide is shown.
eled with the well-known coupled-mode theory, as explained in detail in Section 3.

3. COUPLED-MODE THEORY FOR THE BENT WAVEGUIDE

The fundamental guided mode and also the leaky mode of the straight waveguide can be calculated with classical mode solvers such as the beam propagation method\(^{10}\) or the Fourier decomposition method.\(^{11,12}\) Once the two modes \(\varphi_{1,2}\) of the straight waveguide are (numerically) calculated, the first two modes \(\varphi_{b1,2}\) of the bent waveguide and their phase constants \(\beta_{b1,2}\) can be analytically determined for any bending radius, at no extra computational cost, as the solution of a simple eigenvalue problem.\(^{1,2}\)

Also, the beat length [Eq. (1)] can be analytically calculated for every bending radius \(R\):

\[
\beta_{b1} - \beta_{b2} = [(\beta_1 - \beta_2)^2 + 4\beta_1\beta_2 c_{12}^2/R^2]^{1/2},
\]

(2)

where \(c_{12}\) is the coupling integral between \(\varphi_1\) and \(\varphi_2\) as defined in Ref. 1 and \(\beta_{1,2}\) are the phase constants of the two straight modes. The determination of the second-order mode, however, requires a particular skill, and it is rather long, especially for weakly confined waveguides.

In the following, an easier, alternative method for the determination of the second straight mode’s characteristics is described. This method takes advantage of the coupled-mode theory and avoids the direct calculation of \(\varphi_2\). The bent waveguide can be considered the coupled structure, the coupled modes are \(\varphi_1\) and \(\varphi_2\), and the uncoupled even and odd modes are \(\varphi_{b1}\) and \(\varphi_{b2}\). According to the coupled-mode theory,\(^{11}\) the phase mismatch between \(\varphi_{b1}\) and \(\varphi_{b2}\) is

\[
\beta_{b1} - \beta_{b2} = [(\beta_1 - \beta_2)^2 + 4\kappa^2]^{1/2},
\]

(3)

where \(\kappa\) is the coupling coefficient that is due to the bending. From this equation and the beat length’s definition [Eq. (1)], the phase constant \(\beta_2\) is easily obtained as

\[
\beta_2 = \beta_1 - \left(\frac{2\pi}{L_B}\right)^2 - 4\kappa^2 \right)^{1/2},
\]

(4)

and, by equating Eqs. (2) and (3), we find coupling coefficient to be

\[
\kappa = \frac{c_{12}\sqrt{\beta_1\beta_2}}{R}.
\]

Note that \(\beta_1, \beta_2,\) and \(c_{12}\) depend only on the waveguide geometry and not on the bending radius, whereas the bent-induced coupling coefficient \(\kappa\) depends on \(R^{-1}\).

The coupled-mode theory also predicts the maximum power transfer between the two coupled modes, indicated with \(d\) in Fig. 2,

\[
d = \frac{4\kappa^2}{(\beta_1 - \beta_2)^2 + 4\kappa^2},
\]

(6)

and hence, by means of Eq. (4), a simple relation between \(L_B\) and \(\kappa\),

\[
\kappa = \frac{\pi}{L_B} \sqrt{d},
\]

(7)

is found, from which

\[
\beta_2 = \beta_1 - \frac{2\pi}{L_B} \sqrt{1 - \frac{d}{L_B}}.
\]

(8)

Equations (5), (7), and (8) allow one to determine the phase constant of the bent modes at any bending radius from the knowledge of \(\beta_1, L_B\), and \(d\) at a single given radius.

Once \(c_{12}\) and \(\beta_2\) are known, all the properties of the fundamental bent mode can be analytically determined for any bending radius. This means that the coefficients \(A\) and \(B\),\(^{1}\) which are the perturbations of the phase constant that are due to the bending as well as the coupling losses, and the bending-induced birefringence can be determined without calculating the bend mode or the first leaky mode of the straight waveguide.

In conclusion, the technique can be summarized in the following few steps:

- Determination of the shape and the phase constant of the fundamental mode \(\varphi_1\) of the straight waveguide by a numerical mode solver;
- Simulation of the propagation of \(\varphi_1\) in a bend with a generic bending radius \(R\) for which radiation losses are small;
- Calculation of the fundamental straight mode’s intensity evolution along the bend by projection of the propagating field over \(\varphi_1\); An oscillating behavior that is similar to the one shown in the upper part of Fig. 2 is obtained;
- Determination of the beat length \(L_B\) and of the beat depth \(d\);
- Calculation of \(\kappa, \beta_2,\) and \(c_{12}\) from the above equations.

The five steps can be repeated for both TE and TM modes.

4. MATCHED BEND

If the length of the bend of the structure shown in Fig. 1 is an integer multiple of \(L_B\), the transition losses vanish, and the output field, attenuated only by the radiation losses, is undistorted and can propagate unaffected in the output straight waveguide. A bent waveguide designed according to this criterion is called a matched bend.

A constant-radius matched bend contains an integer number \(m\) of beat lengths; therefore a precise relation between the bending radius \(R\) and the angle \(\theta\) exists. By means of Eqs. (1) and (2) and forcing \(R\theta = mL_B\), the optimum bending radius is

\[
R = \left(\frac{(2\pi m/\theta)^2 - 4\beta_1\beta_2 c_{12}^2}{\beta_1 - \beta_2}\right)^{1/2},
\]

(9)

where the integer \(m\) indicates the length of the bend in units of beat length \(L_B\). In the more general case of matched bends with variable bending radius or variable waveguide width or both, the same procedure leads to an integral equation that relates \(R\) to \(\theta\),

\[
\int_{-\theta/2}^{\theta/2} [R^2(\beta_1 - \beta_2)^2 + 4\beta_1\beta_2 c_{12}^2]^{1/2}d\alpha = 2m\pi,
\]

(10)
where \( a \) is the angular position along the bend, \( R \) depends on \( a \), and \( \beta_1, \beta_2 \), and \( c_{12} \) depend on \( a \) if the waveguide changes its width along the bend. For a given bending angle \( \theta \), the proposed model states that only a discrete number of optimum bending radii exist. For these radii, the excitation of the radiative field \( w_2 \) at the output junction B is theoretically eliminated thanks to the perfect mode matching between the two guiding structures.

The diagram shown in Fig. 3 is obtained from Eq. (9) and refers to the ridge waveguide considered in Section 2. This diagram shows the values of \( R \) and \( \theta \) that satisfy the matched bend condition up to \( m = 8 \) for TE modes, and it permits the correct design of the bend. Each line corresponds to a \( m \)-beat-length long bend. In the diagram the limits are mainly given by the losses of the waveguide, which impose the minimum bending radius. For the considered ridge waveguide, the attenuation is 0.1 dB/rad for \( R = 2 \) mm and 0.4 dB/rad for \( R = 1 \) mm. The parameters are \( \beta_1 = 5.9185 \, \mu \text{m}^{-1} \), \( \beta_2 = 5.889 \, \mu \text{m}^{-1} \), and \( c_{12} = 1.4533 \, \mu \text{m} \) at a wavelength of 1550 nm.

For large bending angles, different solutions are possible, and small bending radii can be used. As an example, a bending angle of 11.4° can be obtained with a bending radius equal to 2, 3, or 4 mm or with even greater radii, whereas the solution for \( m = 1 \) is too lossy. Under the hypothesis that both bent modes undergo a negligible attenuation, there are no differences between these solutions. However, a small bending angle is more difficult to realize with a matched bend. To the left of the \( m = 1 \) curve, in fact, no solutions are possible, and curves with small bending angles require large bending radii. As an example, a bending angle of 1° would require a radius of 13 mm. However, in such a smooth bend, the excitation of the leaky mode, given by Eq. (6), is very weak, and a smaller bending angle is more difficult to realize with a matched bend. To the left of the \( m = 1 \) curve, no solutions are possible, and curves with small bending angles require large bending radii. As an example, a bending angle of 1° would require a radius of 13 mm. However, in such a smooth bend, the excitation of the leaky mode, given by Eq. (6), is very weak, and a smaller bending angle is more difficult to realize with a matched bend. To the left of the \( m = 1 \) curve, no solutions are possible, and curves with small bending angles require large bending radii. As an example, a bending angle of 1° would require a radius of 13 mm. However, in such a smooth bend, the excitation of the leaky mode, given by Eq. (6), is very weak, and a smaller bending angle is more difficult to realize with a matched bend. To the left of the \( m = 1 \) curve, no solutions are possible, and curves with small bending angles require large bending radii. As an example, a bending angle of 1° would require a radius of 13 mm. However, in such a smooth bend, the excitation of the leaky mode, given by Eq. (6), is very weak, and a smaller bending angle is more difficult to realize with a matched bend. To the left of the \( m = 1 \) curve, no solutions are possible, and curves with small bending angles require large bending radii. As an example, a bending angle of 1° would require a radius of 13 mm. However, in such a smooth bend, the excitation of the leaky mode, given by Eq. (6), is very weak, and a smaller bending angle is more difficult to realize with a matched bend. To the left of the \( m = 1 \) curve, no solutions are possible, and curves with small bending angles require large bending radii. As an example, a bending angle of 1° would require a radius of 13 mm. However, in such a smooth bend, the excitation of the leaky mode, given by Eq. (6), is very weak, and a smaller bending angle is more difficult to realize with a matched bend. To the left of the \( m = 1 \) curve, no solutions are possible, and curves with small bending angles require large bending radii. As an example, a bending angle of 1° would require a radius of 13 mm. However, in such a smooth bend, the excitation of the leaky mode, given by Eq. (6), is very weak, and a smaller bending angle is more difficult to realize with a matched bend. To the left of the \( m = 1 \) curve, no solutions are possible, and curves with small bending angles require large bending radii. As an example, a bending angle of 1° would require a radius of 13 mm. However, in such a smooth bend, the excitation of the leaky mode, given by Eq. (6), is very weak, and a smaller bending angle is more difficult to realize with a matched bend. To the left of the \( m = 1 \) curve, no solutions are possible, and curves with small bending angles require large bending radii. As an example, a bending angle of 1° would require a radius of 13 mm. However, in such a smooth bend, the excitation of the leaky mode, given by Eq. (6), is very weak, and a smaller bending angle is more difficult to realize with a matched bend. To the left of the \( m = 1 \) curve, no solutions are possible, and curves with small bending angles require large bending radii.
straight section only the fundamental mode exists, and hence the length \( L \) can be arbitrary. By means of Eq. (9) or the diagram shown in Fig. 3, every S bend is easily designed. More generally, however, it is possible to demonstrate that a matched S bend can also be realized with two unmatched bends. This is not really advisable as the leaky mode \( \varphi_2 \) excited by the first unmatched bend must propagate in the straight section \( L \) to the second bend, and this can give some additional losses if the waveguide is weakly confined.

The geometrical dimensions \( R_1, \theta_1, R_2, \theta_2 \), and \( L \) are determined from the transmission matrix of the S bend. We start by calculating the transmission matrix of the straight–bend discontinuity. This matrix was derived in Ref. 1 and is formed by the eigenvectors of the eigenvalue problem there defined. According to the coupled-mode theory discussed in Section 2, the forward transmission matrix \( J \) of the junction is

\[
J = \begin{bmatrix}
\sqrt{1-a_2^2} & \pm a_2 \\
\mp a_2 & \sqrt{1-a_2^2}
\end{bmatrix},
\]

where \( a_2 = c_{12} \beta_2 / [R(\beta_1 - \beta_2)] \) is the amplitude of the second straight mode calculated for the desired bending radius and the sign indicates the direction of the bend, right or left. Clearly, the bend–straight discontinuity is described by the matrix \( J^{-1} = J^T \).

The transmission matrices both of the bends \( P_{b_j} \) and of the straight section \( P_L \) are diagonals with elements \( \exp(-j\beta_j R) \) and \( \exp(-j\beta L) \), respectively. The transmission matrix \( T_s \) of the whole structure relates the complex amplitudes of the incoming straight modes to the complex amplitudes of the outgoing straight modes and can be simply calculated by back multiplying the various transmission matrices as

\[
T_s = a_2^T P_{b_2} J_2 P_L J_1^T P_{b_1} J_1.
\]

The matched S-bend condition is simply given by the vanishing of the element \( T_s(2, 1) \) and \( T_s(1, 2) \). This condition and the constraints both on the minimum bending radius and on the geometrical dimensions allow one to calculate \( R_1, \theta_1, R_2, \theta_2 \), and \( L \) to obtain the matched S bend.

As an example, consider the S bend of Fig. 5, realized with the ridge waveguide and with \( R_1 = 3 \, \text{mm} \), \( \theta_{1,2} = 10^\circ \), \( R_2 = 2.04 \, \text{mm} \), and \( L = 60 \, \mu\text{m} \). The separation is \( s = 86.4 \, \mu\text{m} \), and two 100-\( \mu \text{m} \)-long straight waveguides are placed at the input and output of the S bend. The evolution of the two straight modes’ intensity along the structure is reported in Fig. 6. At the output of the first bend, which is unmatched, 4% of the leaky mode is present. The second bend, however, reconstructs the fundamental bend mode, and, at the output of the S bend, the intensity of the leaky mode \( \varphi_2 \) is lower than \(-40 \, \text{dB} \). Total losses of the structure are only 0.03 dB.

This example shows the potentiality of the coupled-mode approach to the circuit analysis. The first leaky mode can propagate quite well over hundreds of micrometers. A much more accurate prediction of the circuit behavior should be obtained by taking such a leaky mode into account in the matrix analysis of a complex circuit as, for example, is carried out in Ref. 13.

6. NUMERICAL EXAMPLES AND DISCUSSION

In this section, three types of waveguide are considered and discussed: a 5.2-\( \mu \text{m} \)-by-5.2-\( \mu \text{m} \) square buried waveguide; a rib waveguide 8.5 \( \mu \text{m} \) wide and 6 \( \mu \text{m} \) thick, with a 2-\( \mu \text{m} \)-thick basement and an upper cladding 4 \( \mu \text{m} \) thick; and the ridge waveguide previously introduced. The index difference is \( \Delta n = 0.69\% \), and the analysis is carried out at the wavelength of 1550 nm. The three waveguides present an attenuation of 0.1 dB/\( \text{rad} \) at bending radii of approximately 5, 3.5, and 2 mm, and sketches of the three structures are shown in Figs. 7 and 8. The rib supports one horizontal mode and two vertical modes.

To stress the importance of the matched bend criterion, we show in Fig. 7 the dependence of the beat depth \( \delta \) with respect to the bending radius for both TE and TM modes. The beat depth was defined in Fig. 2. For very large radii of curvature the excitation of the second mode is quite weak, and the criterion can be ignored. The second-mode excitation, however, increases as \( R^{-2} \) and becomes rapidly unacceptable.
The diamonds on the various curves indicate the radii for which the radiation losses of the fundamental bent mode are equal to 0.1 dB/rad. From the figure it can be noted that, for a given bending radius, the ridge waveguide is the less critical one from this point of view as the fundamental mode is less affected by the bending. For a given attenuation, instead, the maximum power converted on the second mode is weaker in the buried waveguide. Moreover, in a square buried waveguide the excitation is identical for both TE and TM modes, whereas in the ridge and in the rib it is different because of the intrinsic asymmetry of the waveguide.

Figure 8 reports the beat-length dependence with respect to the bending radius, both for TE and TM modes. The dependence is very weak, especially for bending radii, which guarantees radiative losses lower than 0.1 dB/rad. Moreover, for small bending radii, for which the matched bend criterion is crucial, the difference between the beat lengths of the TE and TM modes decreases.

It is natural, at this point, to look at some considerations about the sensitivity of a matched bend to the wavelength, the bending radius, or the geometrical perturbations of the waveguide dimensions. Going back to Figs. 2 and 6, we see it is evident that there are sections along the bend in which all the power is carried by the first straight mode only. Such sections possess the minimum sensitivity to a perturbation of the beat length. If \( L_B \) changes by a few percentages, the excitation of the second mode remains negligible.

Also, the dependence on both the waveguide dimensions and the wavelength is quite weak, mainly owing to the same reason discussed above. As an example, the beat length of the previously considered ridge waveguide curved at 2 mm changes by \( \pm 2\% \) when the wavelength changes by \( \pm 30 \) nm around 1550 nm. This small dependence directly appears from Eq. (1) and can be written as

\[
L_B = \frac{\lambda}{(n_{\text{eff},b1} - n_{\text{eff},b2})},
\]

with the difference between \( n_{\text{eff},b1} \) and \( n_{\text{eff},b2} \), i.e., the effective indices of the two bent modes, which is independent of the wavelength of a first-order approximation. This is valid in general, and hence, for any waveguide, \( \frac{\Delta L_B}{L_B} = \frac{\Delta \lambda}{\lambda} \).

The beat-length dependence on the geometrical dimensions and on the index contrast is related to the type of waveguide, but it is generally weak. A variation of \( \pm 0.2 \mu m \) of the width of the ridge waveguide, for example, induces a variation of only \( \pm 4\% \) on \( L_B \). Buried and rib waveguides are even less critical, and the beat-length dependence on the index contrast is much weaker.

The low sensitivity to the technological parameters is another advantage of the matched bend criterion. A standard mask for lithography, for example, shows a resolution around the nominal dimension of \( \pm 0.1 \mu m \), and the impact on a matched bend is totally negligible. This is not the case for another typical solution used for reducing bend losses: the lateral offsets.\(^3\)\(^-\)\(^5\) The principle of lateral offsets is based on the alignment of the field peaks of both the straight and the bent waveguide modes. The formula proposed by Kitoh et al.\(^3\) is valid only for very weakly confined waveguides, and it is not useful for the three waveguides considered above. Alternatively, Hirano et al.\(^4\) proposed the determination of the optimum lateral offset to avoid the excitation of the first-order bent mode. The optimum lateral offset \( \delta \) has been calculated in Ref. 4 as

\[
\delta = R^{-1}(\beta_1 - \beta_2)^{-2}
\]

for multimode waveguides, but the same formula can be applied to single-mode waveguides by use of the leaky-mode propagation constant \( \beta_2 \) given by Eq. (8).

In principle, an optimum lateral offset reduces the bending losses as the lossy mode \( \varphi_{b2} \) is weakly excited. Although this is true for some waveguides, it is not a general statement. Figure 9, for example, shows the total losses of a straight–bend–straight structure of length \( L_B \) for the three considered waveguides, against the lateral offset. Both TE and TM modes are considered. The bending radii are 5, 3.5, and 2 mm for the buried, rib, and ridge waveguides. According to Ref. 4 and Eq. (8), the optimum lateral offsets \( \delta \) should be equal to 0.39, 0.41, and 0.5 \( \mu m \), respectively. From Fig. 9 it is evident that the lateral offset is correctly estimated only for the buried waveguide. Moreover, although both the buried and the rib waveguides receive an advantage from a lateral offset, the ridge waveguide does not present, in practice, any optimum offset that reduces the losses and the output field distortion. The losses indicated for a null lateral offset correspond to the matched bend solution. Unmatched
bends are more lossy in any case, with or without the lateral offset.

Finally, a lateral offset requires highly accurate fabrication parameters, which are not usually compatible with classical lithography and etching processes, and it is not always applicable, as in the case of diffused waveguides. The sensitivity of the losses to the lateral offset is quite high, and even a small error or variation of the optimum value can increase both the losses and, especially, the distortion of the output field. In particular, the optimum lateral offset has a higher wavelength sensitivity, which is \( \Delta \delta / \delta = 2 \Delta \lambda / \lambda \).

As previously mentioned, the presence of the first leaky mode is harmful for a large number of components. Apart from an additional loss, \( \phi_2 \) can compromise the performance, for example, of a Y branch, a MMI coupler, or simply another bend in the cascade. As a first example, we consider a 1 × 2 buried MMI coupler 30 \( \mu \text{m} \) wide and 1595 \( \mu \text{m} \) long. The waveguide dimensions are 5.2 \( \mu \text{m} \) by 5.2 \( \mu \text{m} \), \( \Delta n = 0.69\% \), and the wavelength is 1550 nm. In Table 1 are reported the normalized output powers \( P_1 \) and \( P_2 \), their imbalance, and the losses for the MMI coupler cascaded to the following five buried structures: a straight waveguide (MMI), a matched S bend without (MSb) and with (OMSb) optimum lateral offsets, and an unmatched S bend without (USb) and with (OUSb) optimum lateral offsets. The S bend consists of two bends with \( m = 1 \) (or \( m = 1/2 \) for the unmatched cases) and \( R = 10 \) mm. All the above devices show approximately the same losses, but if the leaky mode \( \phi_2 \) is present at the output of the S bend, then the imbalance is severely affected. Note from Fig. 7 that with a bending radius equal to 1 cm the excitation of the leaky mode is lower than 1%. With reference to the solution with the lateral offsets, the higher residual leaky mode that is present at the output of the S bend produces a higher imbalance (OMSb) with respect to the MSb case. Moreover, in the case of the unmatched S bend, the optimum lateral offsets reduce both the losses and the imbalance with respect to the USb case, but the performance remains far from the MSb case.

As a last example, a Y branch preceded by an S bend is considered. To stress the importance of the criterion, we consider a diffused waveguide, 6 \( \mu \text{m} \) wide and 3.3 \( \mu \text{m} \) high, with an index contrast of only 0.2\%. Though the excitation of the second mode is very weak, its effect on the power-splitting ratio of the Y branch can be unacceptable. The lateral offset is not feasible in this technology.

Figure 10(a) shows the calculated optical field pattern propagating in the S bend followed after 100 \( \mu \text{m} \) of straight waveguide by the Y branch. The S bend is made up of two matched bends with \( R = 5 \) cm and \( \theta = 0.5^\circ \), and the Y branch is very long to avoid any field distortion and loss. As is evident from the figure, the optical field at the output of the matched S bend and in the subsequent straight waveguide section is undistorted. As a consequence, the fields at the Y-branch output are undistorted and balanced within 1%. Figure 10(b) is included for comparison with Fig. 10(a). In this figure an S bend made up of unmatched bends with \( \theta = 0.25^\circ \) and \( R = 5 \) cm is used. The optical fields along the structure are visibly distorted, resulting in distorted and unequal optical fields in the two output waveguide sections. Although the losses are not affected, the imbalance is now equal to 36.8\%.

These two simple examples enhance the importance of avoiding the excitation of the second-order mode. The impairments are even stronger in power splitters with many branches, in star couplers, in switches and modulators in which the extinction ratio and the cross talk cannot be lowered as necessary, and also in many other optical circuits. We believe that the matched bend, eventually joined to lateral offsets if necessary and feasible, can substantially improve both design and performance of a large number of integrated optics circuits.

7. CONCLUSION

In this paper the propagation in a bent waveguide has been described with the coupled-mode theory. On the
basis of the previously proposed description,\textsuperscript{1,2} if the propagating field in the bend is described with two straight modes, a periodic exchange of power between these two modes is observed. Such coupling is induced by the bending. The coupling coefficient and the beat length have been defined, and a simple method for their determination is described. In the case of weak radiation losses, if the length of the bent waveguide is a multiple of the beat length, only the fundamental straight mode is present at the output of the bend. A bend that satisfies such a requirement is called a matched bend. This new concept of the matched bend has been discussed with a large number of examples and comparisons.

The matched bend is just a bend whose curvature radius is accurately chosen. The bending radius depends on the shape and dimensions of the waveguide, and for a given bending angle there exists a discrete number of bending radii that satisfy the matched bend criterion. The bending radius or the bending angle can be analytically determined.

With respect to the lateral offset, matched bends are easier to realize and more robust both to fabrication tolerances and to wavelength. In any case, the matched bend condition guarantees an excellent behavior of the bend. In some cases, in addition to the matched bend condition, a lateral offset can further reduce the losses and the output-field distortion.

The advantage of using matched bends is evident in a large number of structures. Generally speaking, matched bends ensure low losses and undistorted fields, two fundamental properties when two or more components are cascaded or, even more important, in the case of components with an optical feedback such as a ring resonator or optical lattice filters.

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